Methodology to calibrate EIOPA curve

Summary

This document aims to summarize the daily calibration procedure for the Solvency II Risk Free Rate zero coupon curve. It detail only EUR other currencies would have different set of parameter and possibly different equations

Contents

Interface

Inputs (daily)

Swap rates values (updated daily) Mid rates of EUR swap, frequency yearly, against 6 month libor

Maturities 1, 2, 3, 4, 5, 6, 7 ,8 ,9, 10, 11, 12, 15, 20 years. Those maturities can be modified once a year but in practice very rarely so.

Outputs (daily)

- Zero Coupon RFR curve: a vector of 150 rates (maturities 1 to 150 years)
- Alpha_Calibrated
- Status of calibration : Success / Fail

If calibration fail it is intended that the previous Zero Coupon RFR Curve be duplicated.

Parameters

The following is only valid for EUR curve.

Symbol	**Name**	**Update**	**Current value for EUR**
LLP	Last Liquid Point	Yearly but rarely	20
CV	Convergence	Yearly but rarely	40

UFR	Ultimate Forward Rate	Yearly and has been so in the past	3.45%
CRA	Credit Risk Adjustment	Daily but we choose to keep it constant	0.10%

Computation

The following is only valid for EUR curve.

Methodology to model the zero coupon curve

The interpolation, where necessary, and extrapolation of interest rates have been developed applying the Smith-Wilson method

Matrix C

Notation	Detail	Value for EUR
M	Numbers of input par Swap	14
Swap(j)	Yield of input par Swap j (j=1 to M) minus CRA	Input values in real format (1.5% is 0.015) - CRA
Matu(i)	Maturity of input par Swap j (j=1 to M)	1, 2, 3, 4, 5, 6, 7 ,8 ,9, 10, 11, 12, 15, 20 It is assumed all maturities are round number of years
N	Number of coupon date used	20
C	Matrix N lines x M column that serve to input the Swaps values Elements C(i,j), i=1 to M, j=1 to N	C(i,j)= Swap(j) if i<Matu(j) 1+Swap(j) if i=Matu(j) 0 otherwise

Wilson Function (cf 9.7 of [1])

Notation	Detail	Value for EUR
T	Maturity of ZC, Vector Nx1	T(i) = i, i=1 to N
alpha	parameter	
H	Heart of Wilson function, matrix NxN	$H_{i,j} = \propto * \min(T_i, T_j) - \frac{e^{-\propto * max(T_i,T_j)}}{2} * \left[e^{-\propto * min(T_i,T_j)} - e^{\propto * min(T_i,T_j)}\right]$
w	parameter	Log(1+UFR)
W	Wilson function, matrix NxN	$W_{i,j} = e^{-w*(T_i+T_j)} * H_{i,j}$
Mu	Vector Nx1	$Mu_i = e^{-w*(T_i)}$
PZC	Vector Nx1 of 1	$PZC_i = 1$
Teta	Matrix MxM	C'*(W*C)
Vega	Vector Mx1	PZC-C*Mu
Zeta	Vector Mx1	$Zeta \quad = Teta^{-1} * Vega$

If Zeta cannot be inversed an error message is sent

Discount Function

Notation	Detail	Value for EUR
T	Time to compute DF	In years >0

X	Temp matrix MxN	$X_{i,j} = C_{i,j} * e^{-w*(T+T_j)} * \left\{ \begin{matrix} \propto * min(T \;\; ,T_j) \\ -\frac{e^{-\propto * max(T \;\; ,T_j)}}{2} * \left[e^{-\propto * min(T \;\; ,T_j)} - e^{\propto * min(T \;\; ,T_j)} \right] \end{matrix} \right\}$
S	Temp Vector Mx1	$S_i = \sum_{j=1}^{N} X_{i,j}$
DF(T)	Discount Factor at T	$= e^{-w*T} + \sum_{i=1}^{M} Zeta_i * S_i$
ZC(T)	Zero coupon at T	ZC(T)=DF(T)^(-1/T)-1

Methodology to calibrate Alpha

The following is a simplification as EIOPA recently changed methodology. They now apply a derivation of the formula instead of finite difference (see [1]). However the numerical impact is slim and do not justify the added complexity.

Computation of forward rate at convergence maturity by finite difference

Notation	Detail	Value for EUR
T	Convergence maturity	60 =Max(60, LLP+Convergeance)
h	Time increment	0.01 years
FwRlog(alpha)	Forward exponential rate for a given alpha	FwRlog(alpha)= -Ln(DF(T+h)/DF(T))/h
FwR(alpha)	Forward rate	FwR(alpha)= exp(FwRlog(alpha)-1

Calibration of Alpha

Notation	Detail	Value for EUR
Alpha_Init	Initial value of alpha	0.1
Alpha_min	minimal value of alpha	0.05
Alpha_max	maximall value of alpha	0.2
UFRlog	Target for FwRlog	Ln(1+UFR)
Tol	Tolerance de calibration	1e-5
Error(alpha)	Value to minimize	Abs(FwRlog(alpha)-UFRlog)
Alpha_calib	Calibrated value of alpha	
UFR_calib	Calibrated value of UFR	FwR(Alpha_calib)

Algorithm

Minimize Error(alpha) under constraint Alpha_min <= alpha <= Alpha_max

Stop when

Error < 0.00001 ➔ Status= Success

Nb_Iteration > Max_Iteration ➔ Status = Fail

Increment in alpha < 1e-9 ➔ status =Fail

Calibration Output

Notation	Detail	Value for EUR
Status	Calibration status	Success / Fail
Alpha_calib	Calibrated value of alpha	If Fail return 0 else return calibrated value of alpha
UFR_calib	Calibrated value of UFR	If Fail return 0 else return FwR(Alpha_calib)

Output if calibration is successful, using alpha_calib and UFR

Notation	Detail	Value for EUR
T(i)	Maturities of Risk Free Rate EUR curve	1,2,….,150
ZC(i)	Zero coupon at T	ZC(i)=DF(T(i))^(-1/T(i))-1

If calibration of alpha fail an error message is sent

EIOPA recommend to use preceding curve in case calibration fails.

Bibliography

[1] RFR Technical Documentation, The methodology to derive EIOPA's risk-free interest rate term structures, EIOPA(2022)0053701 Supervisory Processes Department PUBLIC, EIOPA-22/547, December 2022 available at https://www.eiopa.europa.eu/tools-and-data/risk-free-interest-rate-term-structures#MonthlyRFRcalculations directory Background material

[2] EIOPA_RFR_20221231_Term_Structures.xlsx available monthly at https://www.eiopa.europa.eu/tools-and-data/risk-free-interest-rate-term-structures#MonthlyRFRcalculations directory Monthly Technical information 2023